## Problem description

Two desirable features in a material model are: 1) the ability to reproduce experimental data in an analysis that corresponds to the experiment and 2) that the material model be stable (positive incremental stiffness). We explore these features for some of the rubber-like material models in ADINA Structures.

For rubber and rubber-like materials, one experiment that can be used to determine material constants is the uniaxial stretching of a sheet experiment schematically shown here.
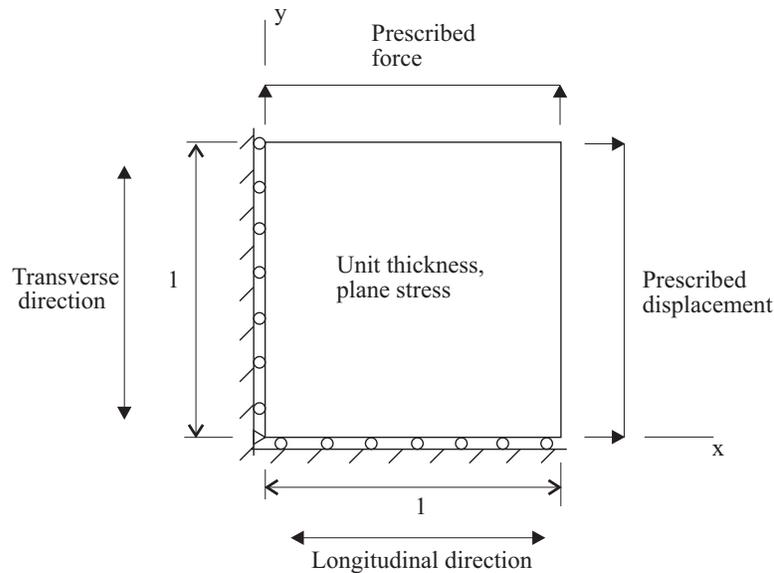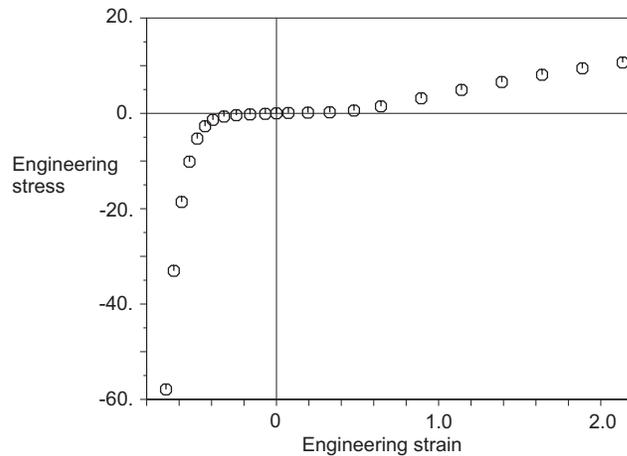

The initial dimensions of the sheet are chosen for numerical convenience. With this choice of initial dimensions, the prescribed displacement can be interpreted directly as a prescribed engineering strain, and the prescribed force can be interpreted directly as a prescribed engineering stress.

Also note that we will subject the sheet to uniaxial compression, and of course an actual sheet would buckle, but our numerical model will not buckle. In practice, uniaxial compression data can be obtained from biaxial tension data.

For the purpose of this primer problem, we assume that the experiments have yielded the stress-strain data shown in the figure on the next page.

This is "constructed" material data, and does not represent any actual rubber or rubber-like material. We have not assigned units to the model or to the data. We have included the engineering stress-engineering strain data points for the material description in the file `prob47_mat.txt`.

We will use three rubber models to describe this material data: Mooney-Rivlin, Ogden and Sussman-Bathe.

In our numerical experiments, we will subject the sheet to uniaxial compression and tension, first without any transverse force, then with a transverse force. The uniaxial compression / tension analysis will demonstrate how well the numerical model reproduces the experimental data, and the transverse force analysis will demonstrate the stability of the numerical model in biaxial tension.

In this problem solution, we will demonstrate the following topics that have not been presented in previous problems:

- Changing the curve-fitting parameters
- Using the Sussman-Bathe material model

Before you begin

Please refer to the Icon Locator Tables chapter of the Primer for the locations of all of the AUI icons. Please refer to the Hints chapter of the Primer for useful hints.

This problem can be solved with the 900 nodes version of the ADINA System.

Much of the input for this problem is stored in file `prob47_1.in`, `prob47_mat.txt`, `prob47_1.plo`, `prob47_2.plo` and `prob47_3.plo`. You need to copy these files from the folder `samples\primer` into a working directory or folder before beginning this analysis.

Invoking the AUI and choosing the finite element program

Invoke the AUI and set the Program Module drop-down list to ADINA Structures.

Model definition

We have prepared a batch file (`prob47_1.in`) which contains all of the model definition except for the material definition.

The loading for the model is as follows. From time 0 to time 40, the x displacement is prescribed and there is no y prescribed force (uniaxial tension/compression conditions). From time 0 to time 7, the x displacement is ramped from 0 to –0.7 (compression); from time 7 to time 14, the x displacement is ramped from –0.7 to 0; from time 14 to time 36, the x displacement is ramped from 0 to 2.2; from time 36 to time 40, the x displacement is ramped from 2.2 to 1.8. The engineering strain is equal to the prescribed displacement.

From time 40 to time 50, the x displacement is held constant at 1.8 and the y prescribed force is increased from 0 to 100.

Notice that constraint equations are used so that the x prescribed displacement is applied at node 1 and the y prescribed force is also applied at node 1.

Choose File→Open Batch, navigate to the working directory or folder, select the file `prob47_1.in` and click Open. The graphics window should look something like the top figure on the next page.

Material definition – Mooney-Rivlin model

Click the Manage Materials icon ![M](M) , click the Mooney-Rivlin button and add material 1. Click the … button to the right of the Fitting Curve field, add Fitting Curve 1, then click the … button to the right of the Simple Tension Curve field and add Curve number 1. Click the Import… button, choose file `prob47_mat.txt` and click OK. In the Define Fitting Curve dialog box, set the Simple Tension Curve to 1 and click OK. In the Define Mooney-Rivlin Material dialog box, set the Fitting Curve to 1 and click Save. Click the Graph button. The new graphics window should look something like the bottom figure on the next page.
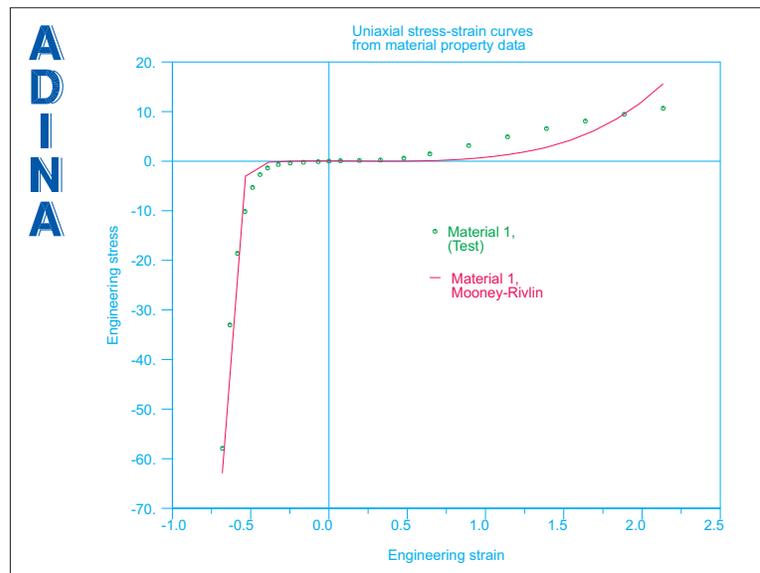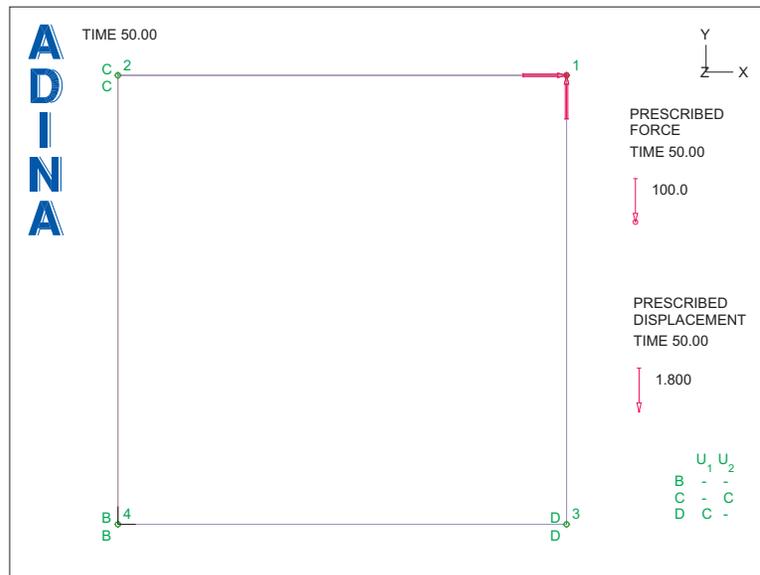
The curve fit does not match the data very well. Close the new graphics window.

Notice that the Generalized Mooney-Rivlin Constants C1 to C9 are all positive. Perhaps we can obtain a closer fit by changing the curve-fitting parameters. Click the … button to the right of the Fitting Curve field, set the Least Squares Solution Method to Gaussian Elimination and click OK. In the Define Mooney-Rivlin Material dialog box, clear the Bulk Modulus field and click Save. When you click the Graph button, the new graphics window should look something like this:
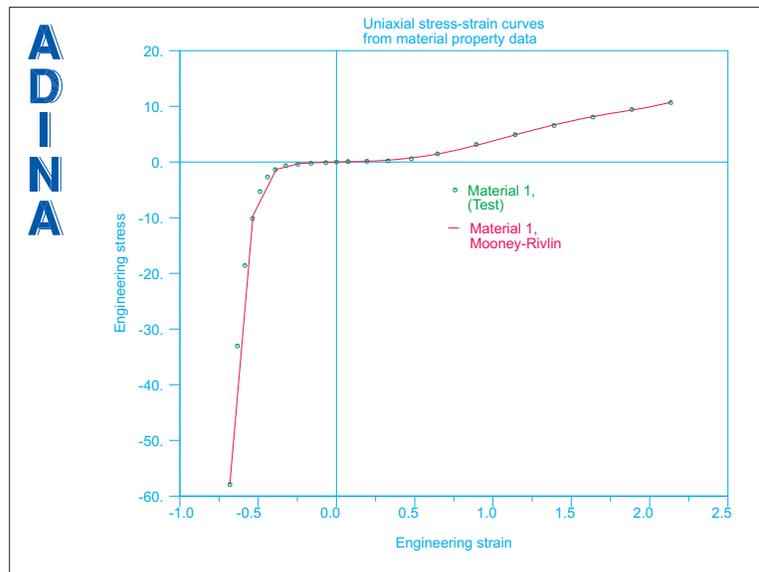

Now some of the Generalized Mooney-Rivlin Constants are negative, but the curve fit is much better. Close the new graphics window, click OK to close the Define Mooney-Rivlin Material dialog box and click Close to close the Manage Material Descriptions dialog box.

Generating the ADINA Structures data file, running ADINA Structures, loading the porthole file

First click the Save icon ![save icon] and save the database to file `prob47`. Click the Data File/Solution icon ![data file icon], set the file name to `prob47`, make sure that the Run Solution button is checked and click Save. When ADINA Structures is finished, close all open dialog boxes. Then set the Program Module drop-down list to Post-Processing (you can discard all changes), click the Open icon ![open icon] and open porthole file `prob47`.

Examining the solution

Uniaxial stress-strain curve: We have put the commands for plotting the uniaxial stress-strain curve in file `prob47_1.plo`. Choose File→Open Batch, navigate to the working directory or folder, select the file `prob47_1.plo` and click Open. The graphics window should look something like this:
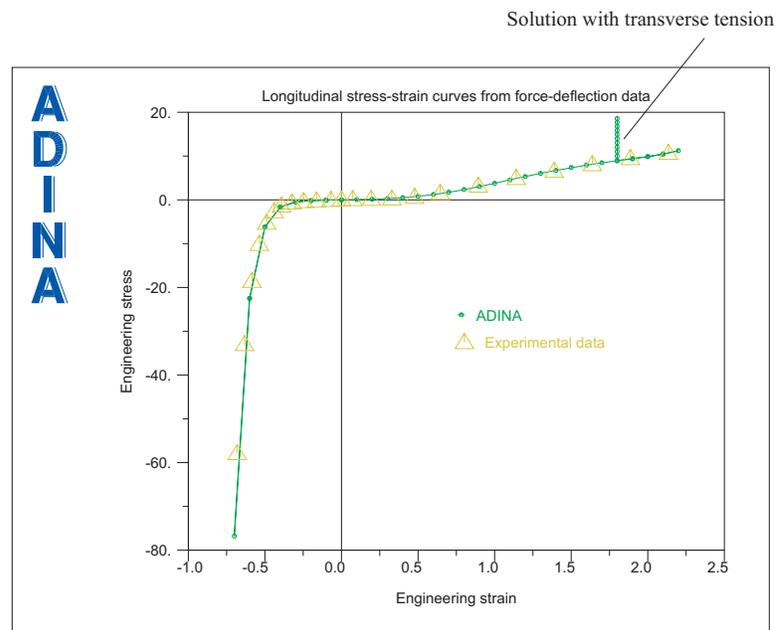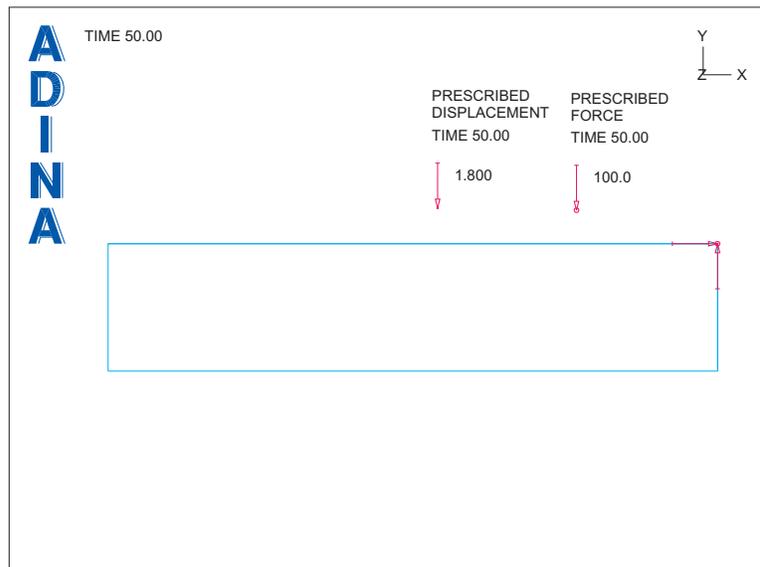

The ADINA Structures solution matches the experimental data closely. The green vertical line shows the solution when we impose a transverse tension.

Transverse stress-strain curve: Let's examine the transverse stress-strain curve more closely. We have put the commands for plotting the transverse stress-strain curve in file `prob47_2.plo`. Choose File→Open Batch, navigate to the working directory or folder, select the file `prob47_2.plo` and click Open. The graphics window should look something like the top figure on the next page.

Transverse stress-strain curves from force-deflection data
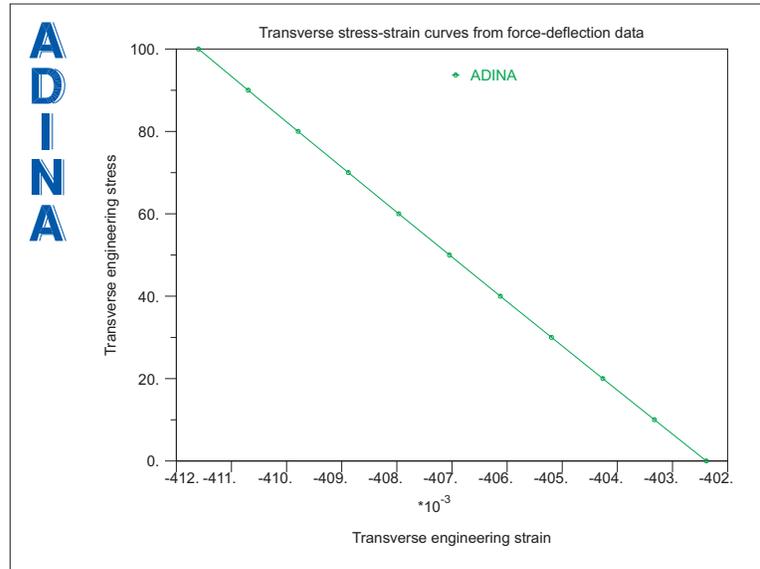
This curve shows that the transverse (y) strain decreases when the transverse stress increases, in other words, that the incremental stiffness in the transverse direction is negative. Let's see if we can show this effect in the mesh. Click the Clear icon [CLEAR], click the Mesh Plot icon and click the Load Plot icon. The graphics window should look something like this:

This plot shows the mesh at the last solution time, when the transverse stress is 100. Click the Fast Rewind icon ◀◀ to display the mesh at solution time 40, when there is no transverse stress, then click the Last Solution icon ▶| to display the mesh at the last solution time. Notice that the mesh shrinks in the transverse (vertical) direction when you click the Last Solution icon ▶| .

This response prediction is non-physical. The Mooney-Rivlin fit used here is unstable (negative incremental stiffness). So, although the Mooney-Rivlin fit shown here fits the uniaxial data very well, this fit cannot describe any real material.

We can also observe that the fit is unstable by carefully examining the output file `prob47.out`. Use a text editor to open this file, and search for the first occurrence of the string

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DIAG ELEMENT (WITH MAX ABS VALUE) OF THE FACTORIZED MATRIX =
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(for this problem, there is only one unknown, so there is only one diagonal element of the factorized matrix). For step 1, the diagonal element of the factorized matrix is 0.76581E+00. Since this is positive, the model is initially stable. (Also note, the printout shows the matrix conditioning at the beginning of the step.)

But at step 3, the diagonal element of the factorized matrix is -0.19822E+01, so the model is unstable for the deformations corresponding to step 2. And at step 41, the diagonal element of the factorized matrix is –0.10605E+05, so the model is unstable for the deformations corresponding to step 40. And this explains why, when we apply a positive transverse stress, we obtain a negative incremental transverse strain.

Changing the material definition – Ogden models

Let's try the Ogden material model. Set the Program Module drop-down list to ADINA Structures (you can discard all changes) and open database file `prob47.idb`.
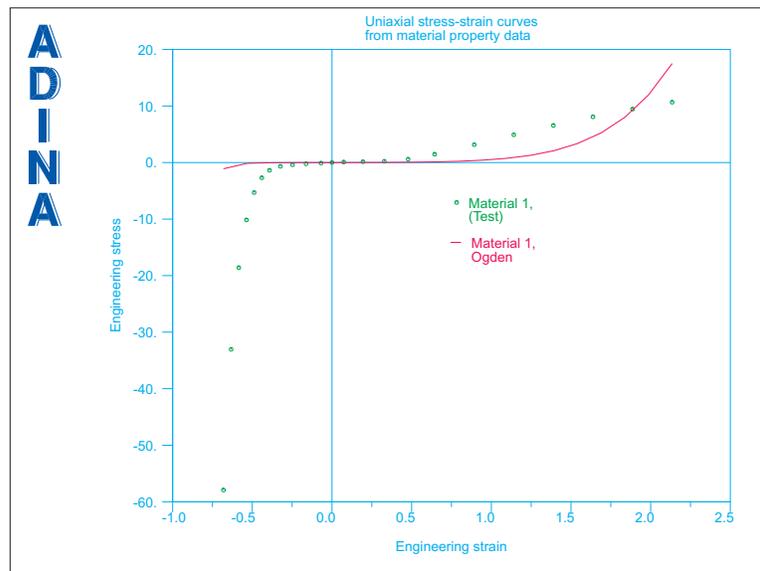
Click the Manage Materials icon **M**, click on row 1 in the table, click the Delete button and click Yes to delete material 1. Now click the Ogden button and add material 1. Click the … button to the right of the Fitting Curve field, set the Least Squares Solution Method to Singular Value Decomposition and click OK. In the Define Ogden Material dialog box, set the Fitting Curve to 1 and click Save. Click the Graph button. The new graphics window should look something like the top figure on the next page.
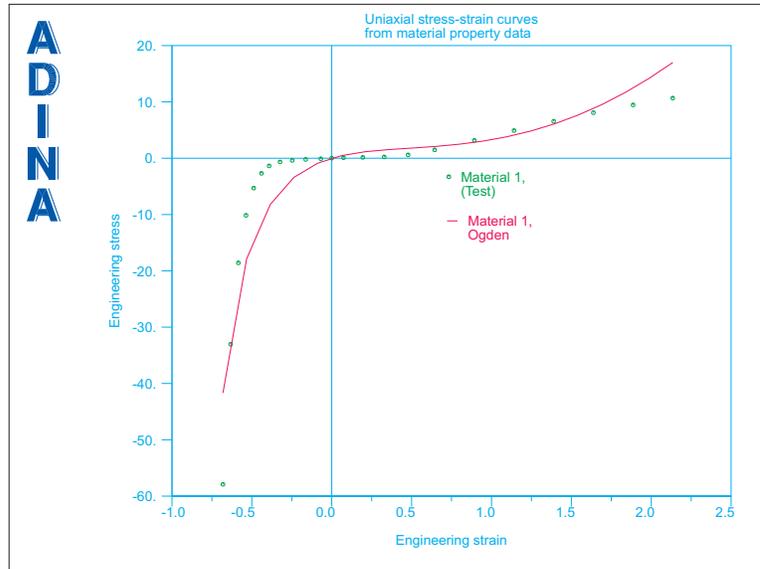
The curve fit does not match the data very well. Close the new graphics window.

Perhaps we can obtain a closer fit by changing the curve-fitting parameters. Click the …
button to the right of the Fitting Curve field, set the Approximation Order to 9 and click OK.
In the Define Ogden Material dialog box, clear the Bulk Modulus field and click Save. When
you click the Graph button, the new graphics window should look something like this:

This fit is even worse.

Perhaps we can improve the fit by using different Alpha constants. Set the Alpha constants as shown in the following table, clear the Bulk Modulus field and click Save.

Term	Alpha
1	0.5
2	-1.0
3	1.0
4	-2.0
5	2.0
6	-3.0
7	3.0
8	-4.0
9	4.0

When you click the Graph button, the new graphics window should look something like this:
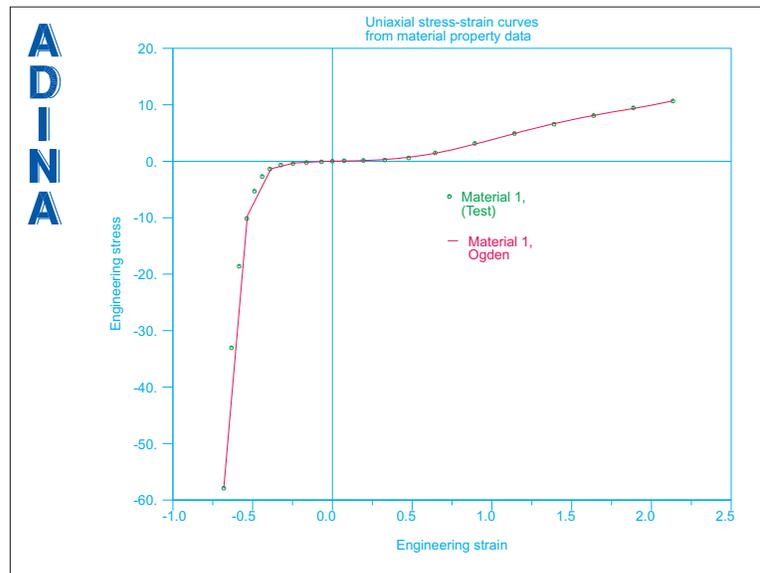
The fit is quite good. Evidently some negative alphas are required for a good fit in this case (the default alphas are 1, 2, 3, …, 9).

Notice that the product of mu i and alpha i is not always positive in this fit, for example the product of mu 2 and alpha 2. When the product of mu i and alpha i is positive for each term, then the material is most likely stable, but this is a "sufficient" condition, not a "necessary" one. For this fit, notice that the product of mu 8 and alpha 8 is positive, and the product of mu
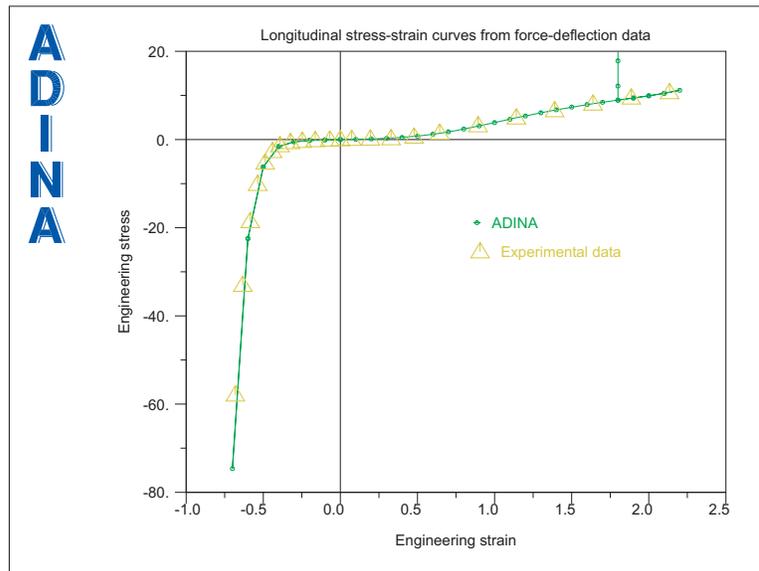
9 and alpha 9 is positive. Since alpha 8 and alpha 9 are the alphas with maximum absolute value, the material will be dominated by these terms when the strains are large.

Close the new graphics window, click OK to close the Define Ogden Material dialog box and click Close to close the Manage Material Descriptions dialog box.

Examining the solution

Save the database, run ADINA Structures, and open the porthole file as described above.
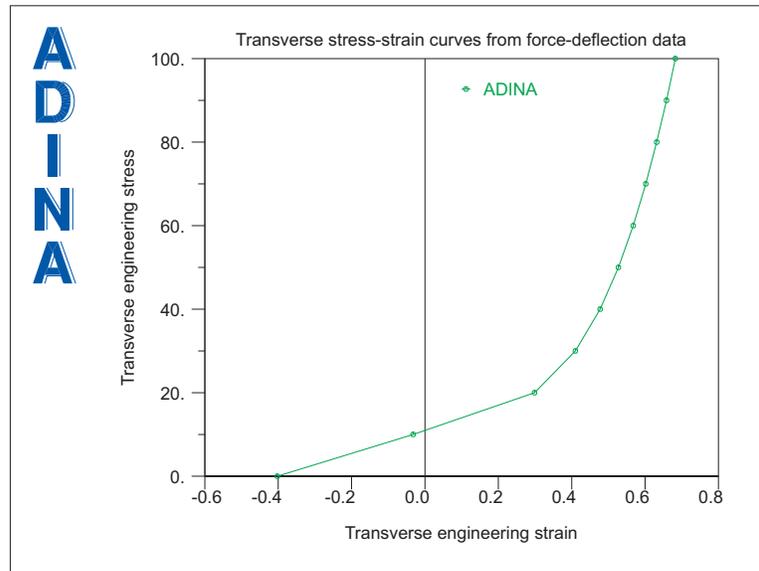
Uniaxial stress-strain curve: Choose File→Open Batch, navigate to the working directory or folder, select the file `prob47_1.plo` and click Open. The graphics window should look something like this:

The ADINA Structures solution matches the experimental data closely.

Transverse stress-strain curve: Choose File→Open Batch, navigate to the working directory or folder, select the file `prob47_2.plo` and click Open. The graphics window should look something like the top figure on the next page.

Transverse stress-strain curves from force-deflection data

This curve shows that the transverse (y) strain increases when the transverse stress increases. So this material description is stable with respect to transverse stresses.

Detail of uniaxial stress-strain curve: For many purposes, the Ogden fit given here is acceptable. But in practical analysis, at a given solution time, there might be parts of the model with very little strain along with parts of the model with large strains. Let's see how this Ogden fit models the data for small strains.

We have put the commands for plotting the uniaxial stress-strain curve for small strains in file `prob47_3.plo`. Choose File→Open Batch, navigate to the working directory or folder, select the file `prob47_3.plo` and click Open. The graphics window should look something like the top figure on the next page.

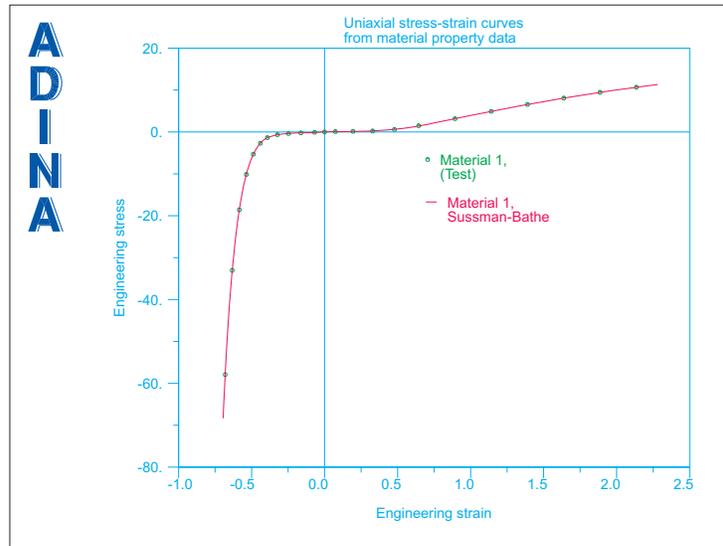This Ogden fit does not fit the small strain data very well.

Longitudinal stress-strain curves from force-deflection data
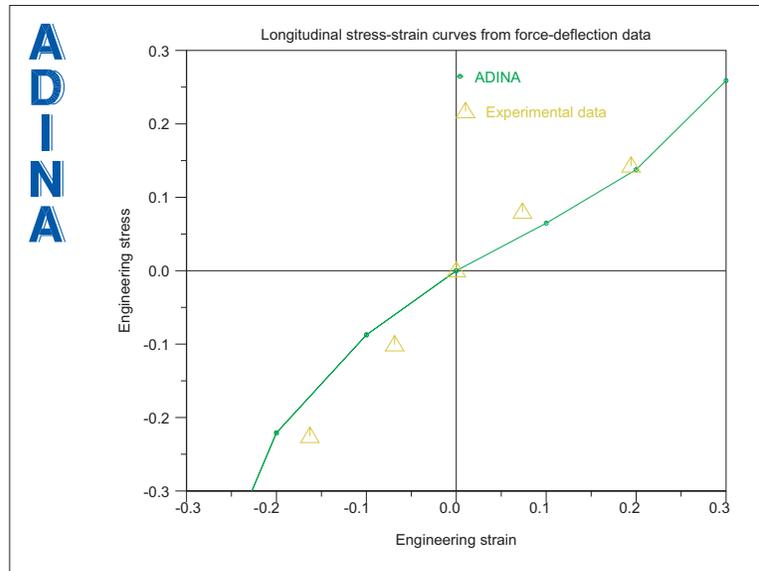
Changing the material definition – Sussman-Bathe model

Let's try the Sussman-Bathe material model. Set the Program Module drop-down list to ADINA Structures (you can discard all changes) and open database file `prob47.idb`.

In the Model Tree, expand the Material entry, right click on 1. Ogden, choose Delete and click Yes. Now click the Manage Materials icon [M], click the Sussman-Bathe button and add material 1. In the Stress-Strain Curve box, check the Reference ID button and set the reference ID to 1. Click the Graph button. The new graphics window should look something like the top figure on the next page.

The fit is extremely good. Close the new graphics window, click OK to close the Define Sussman-Bathe Material dialog box and click Close to close the Manage Material Descriptions dialog box.
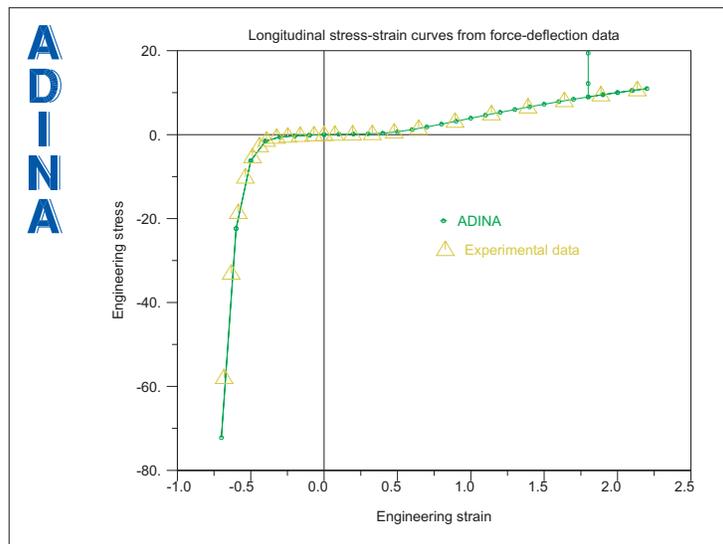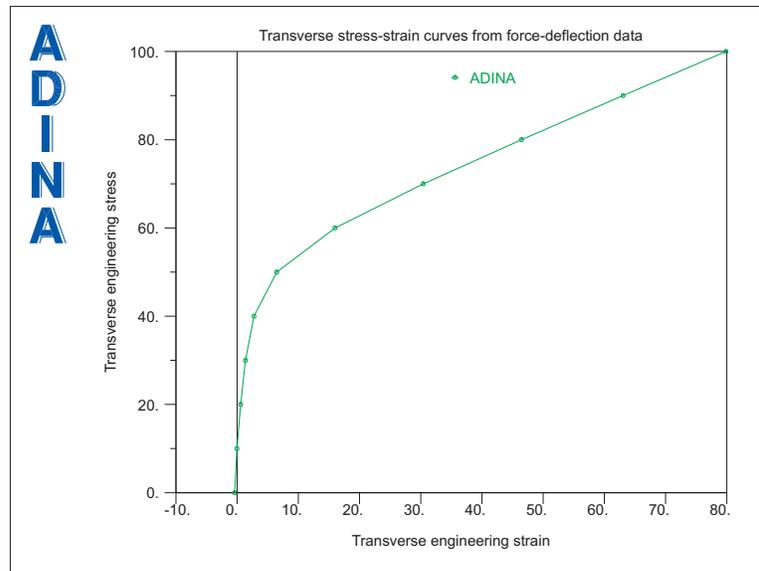
Examining the solution

Save the database, run ADINA Structures, and open the porthole file as described above.

Uniaxial stress-strain curve: Choose File→Open Batch, navigate to the working directory or folder, select the file `prob47_1.plo` and click Open. The graphics window should look something like this:

The ADINA Structures solution matches the experimental data closely.

Transverse stress-strain curve: Choose File→Open Batch, navigate to the working directory or folder, select the file `prob47_2.plo` and click Open. The graphics window should look something like this:


This curve shows that the transverse strain increases when the transverse stress increases. So this material description is stable with respect to transverse stresses. But the transverse stress-strain curve is quite different than that from the Ogden fit for large transverse stresses. (For smaller transverse strains, the Sussman-Bathe and Ogden materials give about the same response prediction, but we do not demonstrate this here.)

Detail of uniaxial stress-strain curve: Let's see how the Sussman-Bathe model fits the data for small strains.

Choose File→Open Batch, navigate to the working directory or folder, select the file `prob47_3.plo` and click Open. The graphics window should look something like the top figure on the next page.
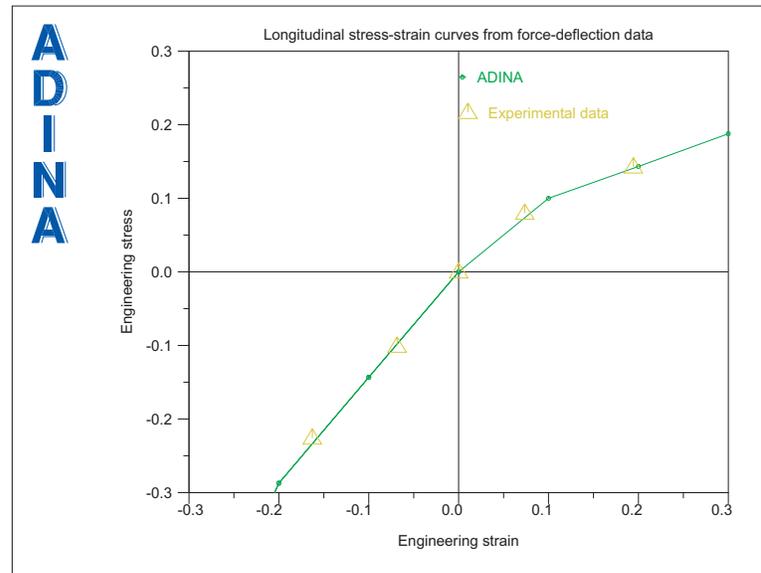
The fit is extremely good for both small strains and large strains.

Exiting the AUI: Choose File→Exit to exit the AUI. You can discard all changes.

Additional notes

Both the Ogden and Sussman-Bathe models are based on the same basic assumption of the separability of the strain energy density

$$W_D = w(e_1) + w(e_2) + w(e_3)$$

see Section 3.8.1.5 of the ADINA Structures Theory and Modeling Guide for details. With this assumption, the material response for all (three-dimensional) strain states can be predicted from the material response for uniaxial tension and compression.

One fundamental difference between the Ogden and Sussman-Bathe models is how the strain energy density term $w'(e)$ is modeled. In the Ogden model, the strain energy density term is modeled using functions with global support (in non-mathematical language, each function spans the entire strain range). In the Sussman-Bathe model, the strain energy term is modeled using functions with local support (each spline segment only spans part of the strain range.) The splines can be adjusted to fit the underlying $w'(e)$ function (obtained from the uniaxial stress-strain data) very well.

In this problem, we use input data in the true (logarithmic) strain range of (-1.14 to 1.14). Hence the input data only defines the $w'(e)$ function only in this strain range. The following graph shows the $w'(e)$ functions of the Ogden and Sussman-Bathe models.
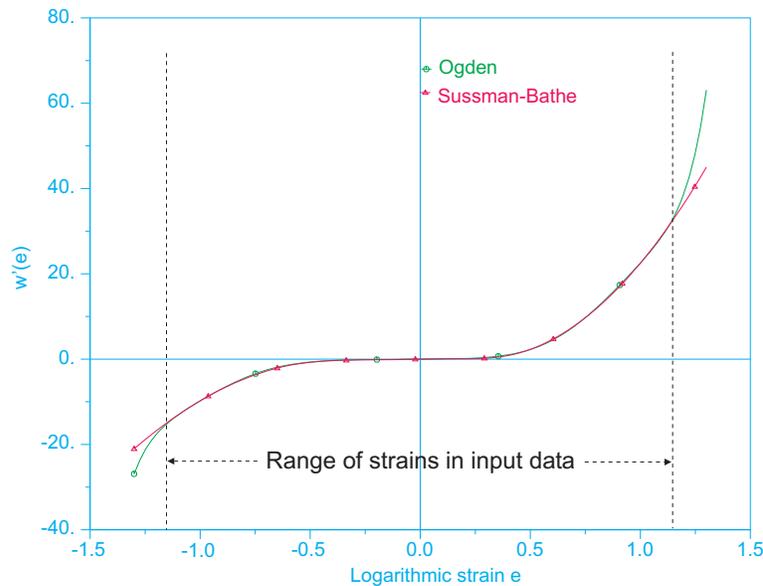
Inside of the strain range of (-1.14 to 1.14), the $w'(e)$ functions of the two models are similar, but outside of this range, the $w'(e)$ functions are different. This explains why both models reproduce the uniaxial stress-strain data, and also why the transverse stress-strain curves from the two models are different. For small transverse strain, all of the strains are in the strain range, so the two models give similar transverse stress/strain curves. But for large transverse strain, the true strain through the thickness becomes smaller than –1.14, so the two models give different transverse stress/strain curves.

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