2.092/2.093 — Finite Element Analysis of Solids & Fluids I Fall '09

Lecture 2 - The Finite Element Analysis Process

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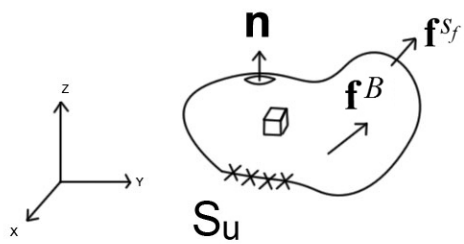
Reading assignment: Chapter 1, Sections 3.1, 3.2, 4.1



We consider a body (solid or fluid) and define the following quantities:

S_u	=	Surface on which displacements, velocities are prescribed
S_f	=	Surface of applied forces or heat fluxes
$\boldsymbol{f}^{S_f}$	=	Forces per unit surface
$\boldsymbol{f}^{B}$	=	Forces per unit volume

We apply to the volume of the body the external loads $\boldsymbol{f}^B$, and to the surface of the body the external loads $\boldsymbol{f}^{S_f}$. Given the volume of the body, the boundary conditions on the surface, the applied loads on the volume and the surface, and the material data, we *solve* for the response of the body. The solution is obtained by satisfying:

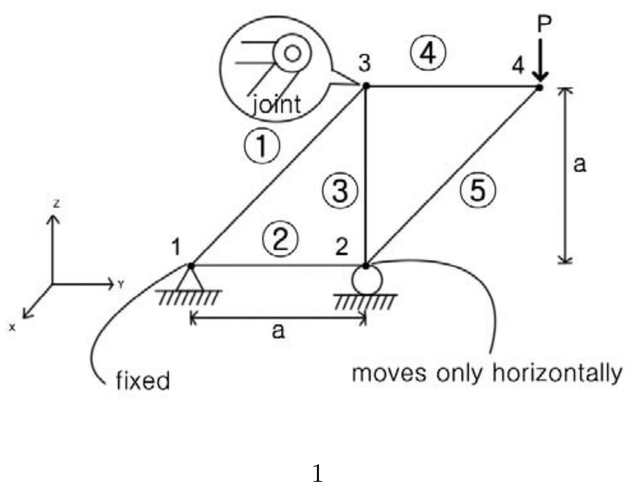
I. Equilibrium of every differential element in the body and on the surface → Momentum equations for fluids

II. Compatibility → Continuity for fluids

III. Stress-strain laws → Constitutive relations for fluids

In linear analysis, the solution is unique.

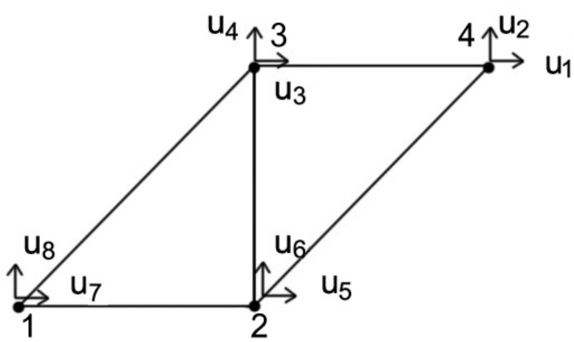
Example: Truss Structure



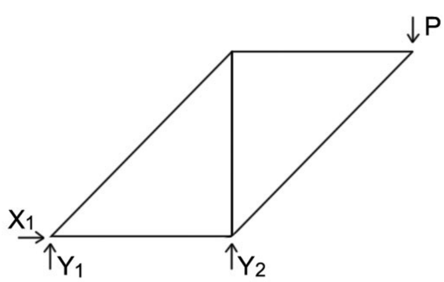
Assume:

- frictionless joints
- weightless bars
- loads applied only at the joints
- joints and elements cannot take moments → truss element does not bend
- concentrated loads only at the joints
- static condition → no vibrations/no transient response
- infinitesimally small displacements, stress below yield stress

So, we perform a linear elastic analysis. Let A = area of each truss element, and E = Young's modulus. Each element has stiffness $\frac{EA}{L_i}$, where L_i is the length of element i. The displacements of the nodes are defined as:

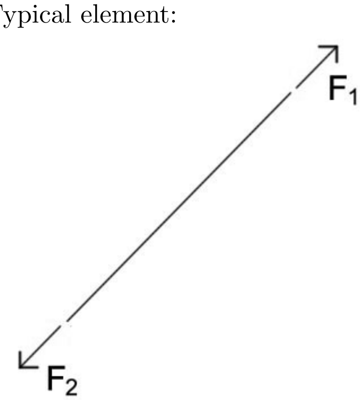


Equilibrium of the structure:



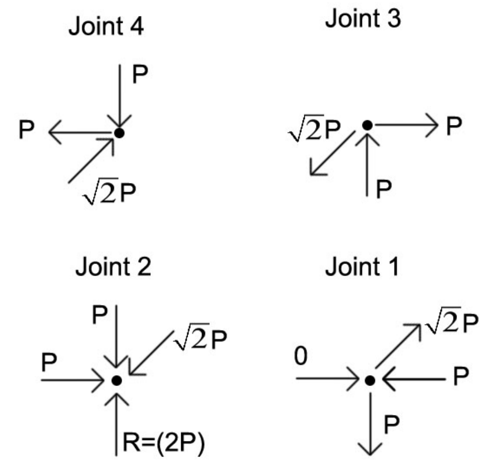
$$Y_2(= R_6) = 2P, \quad Y_1(= R_8) = -P, \quad X_1(= R_7) = 0$$

Typical element:

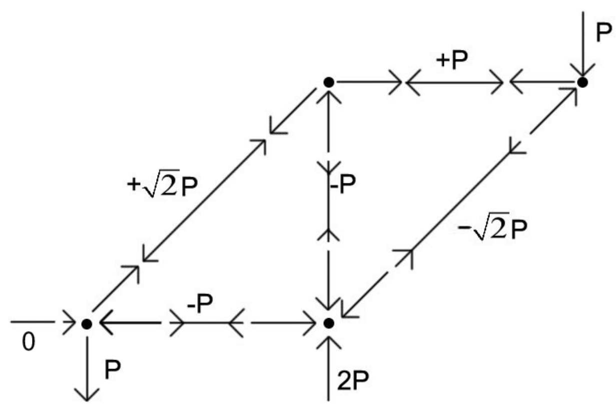


F_1 and F_2 have the same magnitude and are in the opposite direction, acting *onto* the element (resulting in tension in the element).

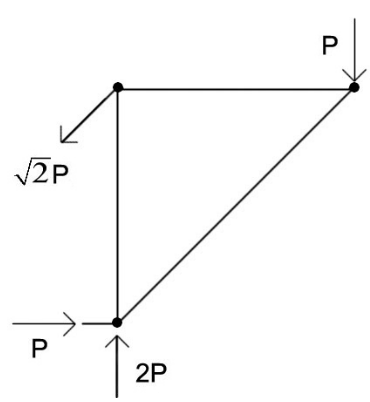
Equilibrium of each joint:



The result is "system equilibrium":



We see that *every element* and *every joint* is in equilibrium, including element forces and reactions and externally applied loads! Also, equilibrium holds for every part of the structure when we include all forces acting on that part. For example, this portion of the structure is in equilibrium:



Actual Solution

$$\boldsymbol{K}\boldsymbol{U} = \boldsymbol{R} \quad \text{(where } \boldsymbol{K} \text{ is the stiffness matrix)}$$

$$\boldsymbol{U} = \begin{bmatrix} u_1 \\ \vdots \\ u_8 \end{bmatrix} \quad , \quad \boldsymbol{R} = \begin{bmatrix} R_1 \\ \vdots \\ R_8 \end{bmatrix}$$

This yields a complete model when the reactions are included. In this case, we have $u_6 = u_7 = u_8 = 0$ and we use only the 5×5 matrix for $\boldsymbol{K}$.

$$\boldsymbol{U} = \begin{bmatrix} u_1 \\ \vdots \\ u_5 \end{bmatrix} \quad , \quad \boldsymbol{R} = \begin{bmatrix} R_1 \\ \vdots \\ R_5 \end{bmatrix}$$

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